

## Discrete Mathematics CS204: Spring, 2008

Jong C. Park Computer Science Division, KAIST Today's Topics Propositions Conjunction Disjunction Negation

## **Logic and Proofs**

6

2



## Logic

- Logic is the study of correct reasoning.
  - It is concerned with whether reasoning is correct.
  - Example
    - A. All mathematicians wear sandals.
    - B. Anyone who wears sandals is an algebraist.
    - C. Therefore, all mathematicians are algebraist.



## **Uses of Logic**

- Proving theorems
- Proving that programs do what they are supposed to do
  - Example
    - We need a program to compute shortest paths between cities.
    - Its input is an arbitrary number of cities and the distances between directly connected cities.
    - Its output: the shortest paths between each distinct pair of cities.
    - How do we know that the program works properly for large input?
    - We need logic, formal or not, to argue that the program is correct.



### Propositions

- Which of the following sentences are either true or false (but not both)?
  - The only positive integers that divide 7 are 1 and 7 itself.
  - Alfred Hitchcock won an Academy Award in 1940 for directing "Rebecca."
  - For every positive integer n, there is a prime number larger than n.
  - Earth is the only planet in the universe that contains life.
  - Buy two tickets to the "Unhinged Universe" rock concert for Friday. (not a proposition)
- A proposition is a sentence that is either true or false, but not both.



## Propositions

#### Definition

- Let p and q be propositions.
- The conjunction of p and q, denoted by  $p \land q$ , is the proposition

#### p and q.

- The disjunction of p and q, denoted by  $p \lor q$ , is the proposition

porq.

Examples

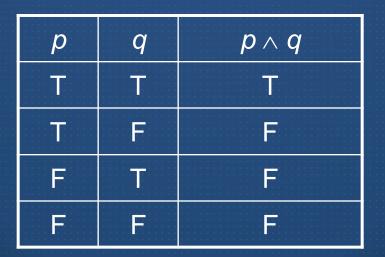
- -p: It is raining; q: It is cold.
- $-p \wedge q; p \vee q$



## Conjunction

#### Definition

#### - The truth value of the proposition $p \land q$ is defined by the truth table



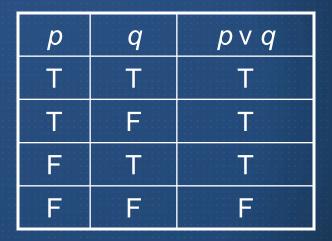
-Note:  $p \land q$  is true only when both p and q are true.



## Disjunction

# Definition The truth table of the proposition p ∨ q is

- Note:  $p \lor q$  is false only when both pand q are false.



- Example
  - -p: It is raining; q: It is cold.
  - $-p \vee q$ : It is raining or it is cold.

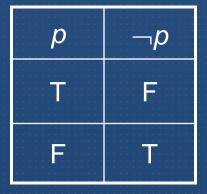


## Negation

#### Definition

## - The negation of p, denoted $\neg p$ , is the proposition not p.

#### The truth table of negation is





## **More Propositions**

Let *p*, *q*, *r* be propositions.
We can form other propositions from *p*, *q*, and *r*, such as ¬*p*∨ *q* ∧ *r*.
Operator precedence

In the absence of parentheses, we first evaluate ¬, then ∧, and then ∨.
With parentheses, we can make the order of evaluation explicit, or change the default order of evaluation, as follows.

- $\neg p \lor (q \land r)$
- $(\neg p \lor q) \land r$





- Propositions
- Conjunction
- Disjunction
- Negation