



Discrete Mathematics

CS204: Spring, 2008

Jong C. Park
Computer Science Division, KAIST



Today's Topics

Propositions

Conjunction

Disjunction

Negation



Logic and Proofs

Logic

- Logic is the study of correct reasoning.
 - It is concerned with whether reasoning is correct.
 - Example
 - A. All mathematicians wear sandals.
 - B. Anyone who wears sandals is an algebraist.
 - C. Therefore, all mathematicians are algebraist.

Uses of Logic

- Proving theorems
- Proving that programs do what they are supposed to do
 - Example
 - We need a program to compute shortest paths between cities.
 - Its input is an arbitrary number of cities and the distances between directly connected cities.
 - Its output: the shortest paths between each distinct pair of cities.
 - How do we know that the program works properly for large input?
 - We need logic, formal or not, to argue that the program is correct.

Propositions

- Which of the following sentences are either true or false (but not both)?
 - The only positive integers that divide 7 are 1 and 7 itself.
 - Alfred Hitchcock won an Academy Award in 1940 for directing “Rebecca.”
 - For every positive integer n , there is a prime number larger than n .
 - Earth is the only planet in the universe that contains life.
 - Buy two tickets to the “Unhinged Universe” rock concert for Friday. (not a proposition)
- A *proposition* is a sentence that is either true or false, but not both.

Propositions

- Definition

- Let p and q be propositions.
- The **conjunction** of p and q , denoted by $p \wedge q$, is the proposition
 p and q .
- The **disjunction** of p and q , denoted by $p \vee q$, is the proposition
 p or q .

- Examples

- p : It is raining; q : It is cold.
- $p \wedge q$; $p \vee q$

Conjunction

- Definition
 - The truth value of the proposition $p \wedge q$ is defined by the truth table

| p | q | $p \wedge q$ |
|-----|-----|--------------|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

- *Note:* $p \wedge q$ is true only when both p and q are true.

Disjunction

- Definition

- The truth table of the proposition $p \vee q$ is

| p | q | $p \vee q$ |
|-----|-----|------------|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

- *Note:* $p \vee q$ is false only when **both** p and q are false.

- Example

- p : It is raining; q : It is cold.
 - $p \vee q$: It is raining or it is cold.

Negation

- Definition

- The negation of p , denoted $\neg p$, is the proposition not p .
- The truth table of negation is

| p | $\neg p$ |
|-----|----------|
| T | F |
| F | T |

More Propositions

- Let p, q, r be propositions.
 - We can form other propositions from p, q , and r , such as $\neg p \vee q \wedge r$.
- Operator precedence
 - In the absence of parentheses, we first evaluate \neg , then \wedge , and then \vee .
 - With parentheses, we can make the order of evaluation explicit, or change the default order of evaluation, as follows.
 - $\neg p \vee (q \wedge r)$
 - $(\neg p \vee q) \wedge r$

Summary

- Propositions
- Conjunction
- Disjunction
- Negation